Lab 5: Sensing Part 2

Part 0: Introduction

In this lab, we will be exploring different filters covered in lecture. We'll be selecting our own cutoff and resonant frequencies for three filters: CR Highpass, RLC band-pass, and CR-RC band-pass.

In order for enough isolation between frequencies for each filter, we want you to target these cutoff or resonant frequencies for each respective circuit:

High Pass	15 kHz or above
RLC band-pass	9 kHz
CR-RC band-pass	2.5-6 kHz

Ultimately, these frequency ranges are **guidelines**: the goal of this lab is to independently light up your LEDs (with little to no overlap).

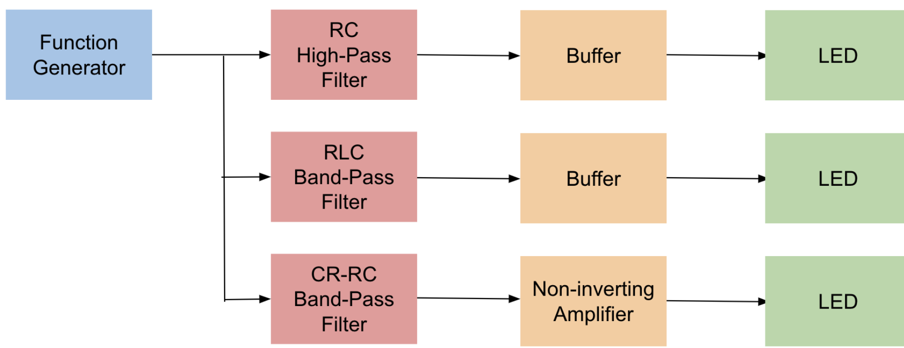


Figure 1: High-level overview of completed lab.

This is a significantly larger circuit than the circuits you have built in previous labs, and will be built on a **separate breadboard**, so **be sure to plan ahead when constructing your circuit, and keep your circuit clean!** You don't want to building the band-pass filter only to find you don't have enough space!

> Sketch on a piece of paper how you plan to allocate the space on your breadboard. After that, you're now ready to build your filters!

Part 1: High-Pass Filter

The first filter we will be building is a first-order RC high pass filter. This is the simplest high pass filter implementation, and can be implemented with a single pole (thus, making it a first order filter). Frequencies above the pole frequency are allowed to pass, and any frequencies below the pole frequency are attenuated. This pole frequency is also commonly referred to as the **cutoff frequency**.

The transfer function of the first order RC highpass filter is as follows:

$$H(j\omega) = \frac{Z_R}{Z_C + Z_R} = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC}$$

Notice that we have a $j\omega RC$ term in the numerator. This means that we have a zero at $j\omega = 0$. **This makes sense... What does the Bode plot for a high pass filter look like? How does a zero affect the Bode plot of a transfer function?**

For more information on high pass filters, refer to Appendix A.

Part 2: RLC Band-Pass Filter

In the second part of this lab, we will be building an RLC band-pass filter. Depending on how sharp the filter is, it can either filter out all frequencies besides one resonant frequency, or pass a band of frequencies. This specific second order band-pass filter is implemented by placing an inductor, capacitor, and resistor in series.

Your circuit should look like this:

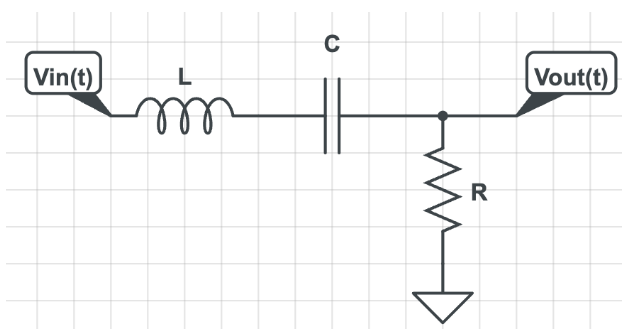


Figure 2: RLC band-pass Filter.

Its transfer function can be written as:

$$H(j\omega) = \frac{\widetilde{V}_{out}}{\widetilde{V}_{in}} = \frac{Z_R}{Z_L + Z_C + Z_R} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

It is obvious that $|H(j\omega)|$ will be equal to 1 if $\omega L = \frac{1}{\omega C}$ (look at the denominator of the final equation for $H(j\omega)$). The frequency where this equation is true is known as the resonant frequency–the frequency at which the circuit naturally oscillates when not driven by an external source. We can derive the formula for the resonant frequency ω_0,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0{}^2 LC = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At resonant frequency, the L and C reactances cancel each other out and create a short circuit, leaving only the resistor. For our RLC band-pass filter, the frequencies around ω_0 will be passed, while frequencies further away from

ω_0 will be attenuated. Intuitively, this makes sense as we showed that $|H(j\omega)|$ was be equal to 1 at the resonant frequency.

This band-pass circuit contains one resonant frequency ω_0 and a characteristic called the Q factor, where for an RLC circuit in series,

$$Q = \frac{\omega_0 L}{R}$$

At a high level, the Q factor represents the 'quality' of RLC filters. The higher the Q factor, the smaller the bandwidth around the resonant frequency, which means that there will be sharper attenuation at the frequency of choice. The Q factor can also be used to determine whether a system is underdamped, overdamped, or critically damped. A higher Q factor corresponds to underdamping, whereas a small Q factor corresponds to overdamping. You can think of the resistor in the circuit as what causes the damping. Energy is dissipated in our resistor, whereas inductors and capacitors both store energy in the form of magnetic and electric fields respectively. Therefore the larger the resistance, the higher the damping of the system (how does this relate to frequency attenuation in our magnitude response?). Please refer to Appendix B for more information about second order filters.

Sanity check question: *Does the resistor value in an RLC filter matter? I.e. how does it impact the response of our RLC band-pass filter?*

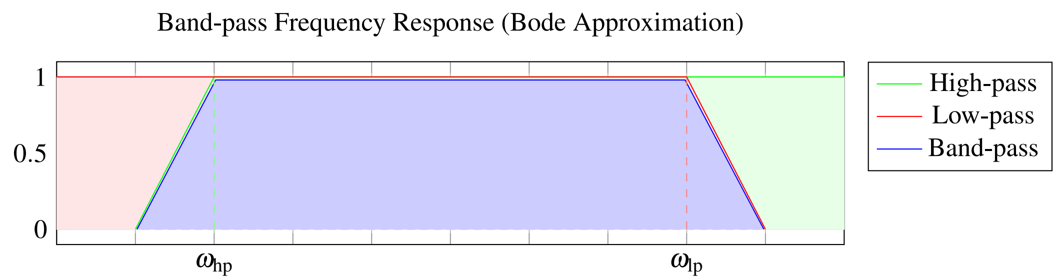
Exercise left for the reader: *The above example uses a RLC circuit connected in series in order of inductor, capacitor, then resistor. What happens when we have the circuit connected in series in order of resistor, inductor, and capacitor? How is this new circuit different from our current one? What happens to the magnitude response? Where is its resonance frequency?*

Part 3: CR-RC Band-Pass Filter

In the final part of this lab, we will be building a second-order band-pass filter by chaining/cascading together a low-pass and high-pass filter.

The band-pass filter has two cutoff frequencies: ω_{hp} and ω_{lp}, which correspond to the high-pass and low-pass cutoff frequencies respectively. **Which of the two cutoff frequencies should be higher?**

The frequency response of the band-pass will look something like this:



Sanity check question: *Does the order of the filters matter? I.e. does it matter whether we chain low-pass into high-pass or vice versa?*

Note that although both are named band-pass filters, the RLC band-pass has a different frequency response from the CR-RC band-pass filter.

Sanity check question: *When might we want to use the RLC Band-pass filter over the CR-RC and vice versa?*

In order to accomplish this, we want the transfer functions of our two filters to multiply. However, we need to be careful: we cannot simply plug the output of one of the filters into the input of the other directly; doing so would "load" the first filter and affect its cutoff frequency because the second filter ends up drawing current from the first. If we want to make sure the transfer functions to multiply and give us the desired band-pass frequency response without affecting each other, we need to somehow isolate the first filter from the second while still passing the output of the first to the input of the second. Here is where buffers come into play.

Buffers

You can think of a buffer as providing an impedance transformation between two *cascaded* circuits. When you observe an undesired loading effect between two circuits, placing a buffer between them changes the load impedance of the first circuit to a very high value and the source impedance of the second to a very low value in accordance with (approximately) ideal op-amp characteristics. As the op-amp does not allow any current to flow into its input terminals, this prevents the second filter from drawing current from the first filter and affecting the frequency response. Instead, the second filter draws its current from the output of the op-amp, which is a replica of the first filter's output due to this being a buffer circuit. This allows you to build very modular circuits easily, without having to do lots of ugly algebra.

By placing a buffer in between our two filters that make up the band-pass filter, *cascading them does not change the transfer functions of the individual circuits* and the overall transfer function of the cascade is simply the product of the transfer functions of the individual circuits. This is why buffers are so useful in filter design.

Your final CR-RC filter should look similar to this.

Figure 3: CR-RC band-pass diagram.

Note that the order of the low-pass and high-pass do not matter. Convince yourself why this is the case.

Part 4: Connecting to LEDs

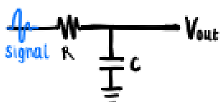
Finally, we want to connect our 3 new filters each to their respective LEDs. However, we cannot directly connect the output of our filters to our LEDs as the LEDs may load our filters (similar to how we could not directly connect the low-pass and high-pass filters together for the CR-RC Band-Pass). Furthermore, the voltage output of the CR-RC band-pass may not be strong enough at the peak to light up fully (why is this the case?). Therefore, like we did in Part 3, we must create a buffer after the output of our filters before driving the LED, and for the CR-RC band-pass in particular, we recommend utilizing a non-inverting amplifier to act as a buffer and provide a gain to increase the voltage output. **This means that you will be building a total of 3 buffers and 1 non-inverting amplifer throughout this lab** Although tedious, it is imperative you do this to get a working color organ!

Appendix A: Derivation of first and second order RC filters

Building Filters

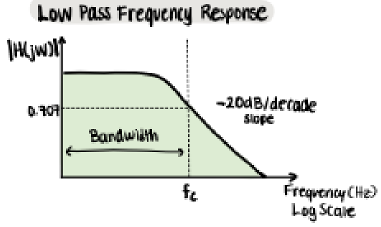
Lowpass Filter

Circuit Schematic



Think: the "gate" C is lower.

Low Pass Frequency Response



$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_C}{Z_R + Z_C} = \tilde{V}_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \tilde{V}_{in} \frac{1}{j\omega RC + 1}$$

$\frac{V_{out}}{V_{in}} = H(j\omega)$ and cutoff frequency is at half power, where $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = 0.707$.

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{1}}{\sqrt{(\omega RC)^2 + (1)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$2 = 1 + (\omega RC)^2$$

$$1 = \omega RC$$

$\omega = \frac{1}{RC}$ angular cutoff frequency

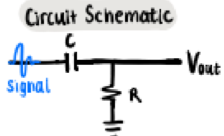
$f_c = \frac{1}{2\pi RC}$ cutoff frequency

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 0$
as $\omega \to 0$, $|H(j\omega)| \to 1$

Everything that is less than f_c gets through. Note that our cutoff isn't clean & perfect because the attenuation is gradual.

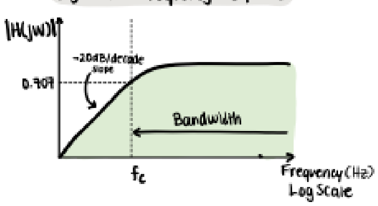
High Pass Filter

Circuit Schematic



Think: the "gate" C is higher.

High Pass Frequency Response



$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_R}{Z_R + Z_C} = \tilde{V}_{in} \frac{R}{\frac{1}{j\omega C} + R}$$

$$|H(j\omega)| = \frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{R^2}}{\sqrt{(\frac{1}{\omega C})^2 + R^2}}$$

$$\frac{1}{2} = \frac{R^2}{(\frac{1}{\omega C})^2 + R^2}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = 2R^2$$

$$\left(\frac{1}{\omega C}\right)^2 = R^2$$

$\omega = \frac{1}{RC}$ angular cutoff frequency

$f_c = \frac{1}{2\pi RC}$ cutoff frequency

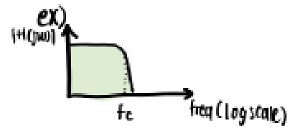
Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 1$
as $\omega \to 0$, $|H(j\omega)| \to 0$

Everything higher that f_c gets through.

Thought: What happens to DC Voltage in a high pass filter?
↳ It gets destroyed, $\omega = 0$!

Thought: How can we make attenuation faster?
↳ multiple filters cascaded. Our transfer functions multiply, making the drop-off faster.
↳ make sure to place a unity gain buffer in between to prevent loading

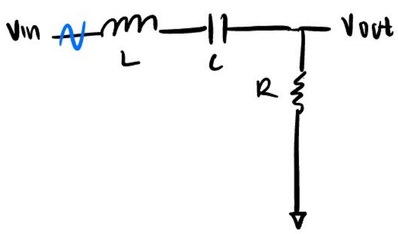
ex)



Appendix B: Derivation of second order filters

2nd Order Filters

RLC Bandpass



$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_R + Z_L + Z_C} = \frac{R}{\frac{1}{j\omega C} + j\omega L + R} = \frac{1}{1 + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Resonant Frequency $\Rightarrow$ when $|H(j\omega)| = 1$

$\Rightarrow$ when $\omega L - \frac{1}{\omega C} = 0$

$\Rightarrow \omega^2 L - \frac{1}{C} = 0$

$\omega^2 = \frac{1}{LC}$

$\omega = \sqrt{\frac{1}{LC}}$

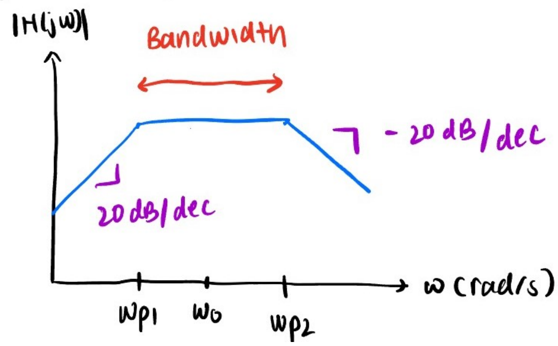
$\Rightarrow$ resonant frequency. $\omega_0 = \sqrt{\frac{1}{LC}}$

Series Damping: $\alpha = \frac{R}{2L}$ (see lecture 3B)

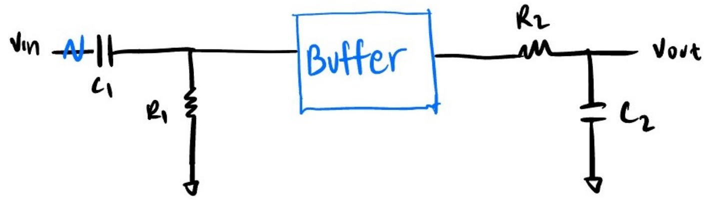
poles: $-\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Q factor: $\frac{\omega_0 L}{R}$

Magnitude Response:



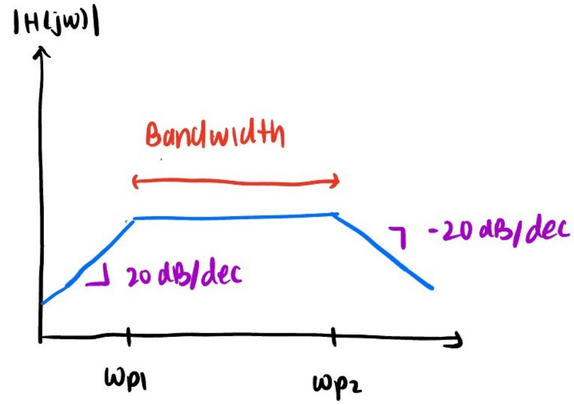
RC Bandpass



$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_{R1}}{Z_{R1} + Z_{C1}} \cdot \frac{Z_{C2}}{Z_{R2} + Z_{C2}} = \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1} \cdot \frac{1}{1 + j\omega R_2 C_2}$$

poles: $\omega_{p1} = \frac{1}{R_1 C_1}$, $\omega_{p2} = \frac{1}{R_2 C_2}$

zeros: $\omega_z = 0$



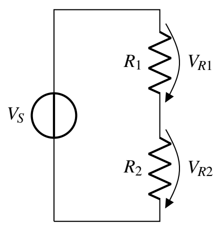
Thought: How do we get +20dB/dec of slope for both bandpass filters?
↳ A zero at $j\omega = 0$ gives us a slope of +20 dB/dec

Thought: The filter we're building in lab is called a notch-pass. Using RLC, is there a way we can make a filter that only attenuates at one specific frequency? (Hint: look up notch-stop).

Appendix C: EECS16A Circuits Cookbook

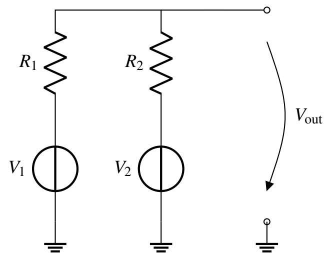
(For Reference: Example Circuits)

Voltage Divider



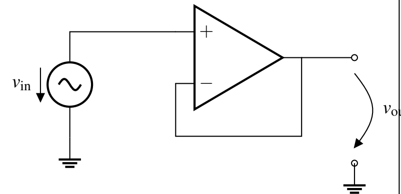
$$V_{R2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Voltage Summer



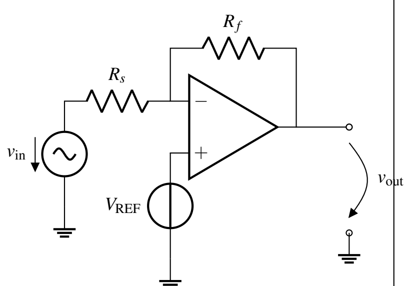
$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



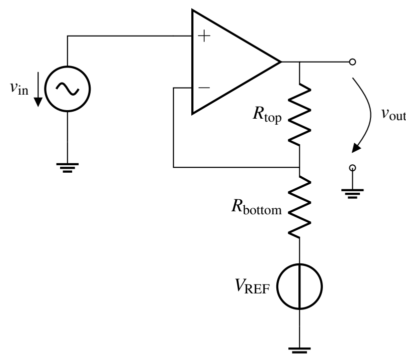
$$\frac{v_{out}}{v_{in}} = 1$$

Inverting Amplifier



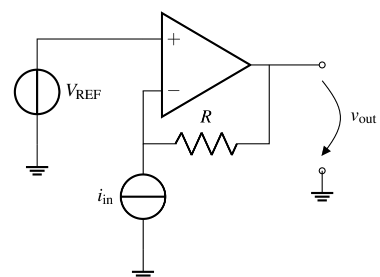
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right) + V_{REF}\left(\frac{R_f}{R_s}+1\right)$$

Non-inverting Amplifier



$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right) - V_{REF}\left(\frac{R_{top}}{R_{bottom}}\right)$$

Transresistance Amplifier



$$v_{out} = i_{in}(-R) + V_{REF}$$

References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.
Sedra, A. and Smith, K. (2015). *Microelectronic Circuits.* 7th ed. New York: Oxford University Press, ch 17.
https://www.electronics-tutorials.ws/wp-content/uploads/2018/05/resistor-res5.gif
https://www.bragitoff.com/wp-content/uploads/2015/09/CapacitorsCheatSheet.png

Written by Mia Mirkovic (2019).
Edited by Kourosh Hakhamaneshi (2020). Version 2.0, 2020.
Edited by Steven Lu (2021). Version 3.0, 2021.
Edited by Megan Zeng, Bozhi Yin (2022). Version 4.0, 2022.
Edited by Jessica Fan (2023). Version 5.0, 2023.
Edited by Jessica Fan, Junha Kim, Ryan Ma (2024). Version 6.0, 2024.